

Tangents, Rates of Change, and Derivatives

Definition: The *tangent line* to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

provided that the limit exists.

If an object moves along a straight line according to an equation of motion $s = f(t)$, where s is the displacement (directed distance) of the object from the origin at time t , then the *velocity* $v(a)$ at time $t = a$ is

$$v(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}.$$

Definition: The **derivative of a function f at a number a** , denoted by $f'(a)$, is

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

if the limit exists.

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a .

The derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$.

Recall the point-slope form for the equation of the line passing through (x_0, y_0) with slope m :

$$y - y_0 = m(x - x_0).$$