MATH 1210 TEST 2 Name:

Each problem is worth 15 points. The bonus is worth 10 points.

1. Find the limit: $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)$

2. Find the limits:

a). $\lim_{x\to 2^-}\frac{x-|x-2|-2}{x^2-4}$

b). $\lim_{x\to 2^+}\frac{x-|x-2|-2}{x^2-4}$

3. If $f(x) = x^2 + 10 \sin x$, show that there is a number c such that $f(c) = 1000$.

4. Find all values of a such that f is continuous on $\mathbb{R}$:

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq a \\ x^2 & \text{if } x > a \end{cases}$$

5. Find $\lim_{x\to\infty} f(x)$ if, for all $x > 1$,

$$\frac{x - x^5 - x^4}{(1-x^2)(3x^3-1)} < f(x) < \frac{e^x - 1210}{3e^x}.$$

6. If $f'(x) = x^3 - 4x$, find:
a). all points where the tangent line to the graph of f is horizontal (justify your answer)

b). on what intervals is f increasing? (justify your answer)

7. Find the derivative of the function $f(x) = x + \sqrt{x}$. Then state the domain of the function and the domain of its derivative.

8. [BONUS] Write down your favorite number (let's call it ζ). Find functions $f(x)$ and $g(x)$ so that $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = \infty$ and $\lim_{x\to 0} [f(x) - g(x)] = \zeta$, your favorite number.