The Fundamental Theorem of Calculus

The FTC, Part 1 If f is continuous on $[a,b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt, \quad a \le x \le b$$

is an antiderivative of f, that is, $g'(x) = f(x)$ for $a < x < b$.

The FTC, Part 2 If f is continuous on the interval $[a,b]$, then

$$\int_a^b f(x)\,dx = F(x)|_a^b = F(b) - F(a)$$

where F is any antiderivative of f, that is, $F\,' = f$.

Notes:

1. We can combine the Chain Rule and FTC1 to get:

$$\left(\int_a^{u(x)} f(t)\,dt\right)' = f(u(x)) \cdot u'(x)$$

2. If both limits of integration contain the variable x then we "break" the integral like this:

$$\int_{u(x)}^{v(x)} f(t)\,dt = \int_{u(x)}^{a} f(t)\,dt + \int_a^{v(x)} f(t)\,dt = -\int_a^{u(x)} f(t)\,dt + \int_a^{v(x)} f(t)\,dt$$

and then use FTC1 combined with the Chain Rule to take derivatives.