The Derivative Function

Definition: The function $f'(x)$ defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is called the **derivative of** f, whenever the limit exists.

Notations: Let $y = f(x)$ be a function. Then its derivative is

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x).$$

Definition: A function f is **differentiable at** a if $f'(a)$ exists. It is **differentiable on an open interval** (a, b) if it is differentiable at every number in the interval.

THEOREM If f is differentiable at a, then f is continuous at a.
The converse of this theorem is not true! There are continuous functions that are not differentiable.

If f is a function then

$$f'' = (f')'$$

is called the **second derivative** of f. Another notation is

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}.$$

The third derivative is

$$f''' = (f'')' = \frac{d^3y}{dx^3}.$$