Continuity

Definition A function f is *continuous at a number* a if $\lim_{x \to a} f(x) = f(a)$.

If f is not continuous at a we say that f is *discontinous* at a.

Definition A function f is *continous from the right at* a if $\lim_{x \to a^+} f(x) = f(a)$ and f is *continuous from the left at* a if $\lim_{x \to a^-} f(x) = f(a)$.

Definition A function f is *continuous on an interval* if it is continuous at every number in the interval.

THEOREM If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

$f + g, \quad f - g, \quad cf, \quad fg, \quad \dfrac{f}{g}$ if $g(a) \neq 0$.

THEOREM The following types of functions are continuous at every number in their domains: POLYNOMIALS, RATIONAL FUNCTIONS, ROOT FUNCTIONS, TRIGONOMETRIC FUNCTIONS, INVERSE TRIGONOMETRIC FUNCTIONS, EXPONENTIAL FUNCTIONS, LOGARITHMIC FUNCTIONS.

THEOREM If f is continuous at b and $\lim_{x \to a} g(x) = b$, then $\lim_{x \to a} f(g(x)) = f(b)$. In other words,

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right).$$

THEOREM If g is conyinuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

THEOREM (THE INTERMEDIATE VALUE THEOREM) Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in the interval (a, b) such that $f(c) = N$.